BHPS Calculations Policy

January 2018

What is Mastery at BHPS?

- High expectations – **all** pupils succeed with the same objective with support and challenge where appropriate
- Differentiation defined by deepening knowledge or supporting/ intervening not different concepts or objectives
- New concepts introduced in the context of a problem, story, picture
- Problems represented with concrete or pictorial resources
- An expectation that children will move through the concrete, pictorial, abstract stages
- A variety of methods taught and calculations represented in different ways to promote understanding. E.g. part whole, bar.
- Explicit teaching of vocabulary
- Effective, precise questioning that regularly assesses where pupils are at
- An emphasis on children explaining their reasoning and calculations clearly
- Use of stem sentences
- Opportunities for pre-teach and post-teach to prepare children and address misconceptions
- Practice and consolidation have a place in lesson design (e.g. warm up with fluency activities to hone skills that will be needed in the lesson)
- Reasoning and problem solving is an entitlement for all abilities
- Intervention at the point of need
- More whole class teaching at KS1 with children being as active as possible with apparatus in their places
- Leading to a variation: 'Do it' (independent and varied fluency in different contexts) 'Secure it' (shift of original problem to different contexts, comparisons of different problems) 'Deepen it' (Explain, justify, prove ideas. – True/False; always/sometimes/never)

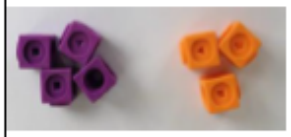
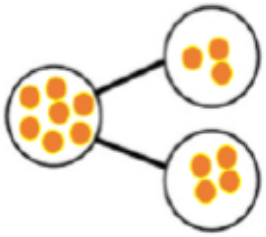
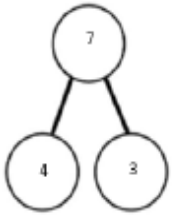
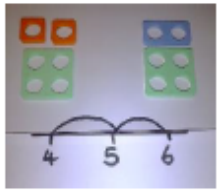
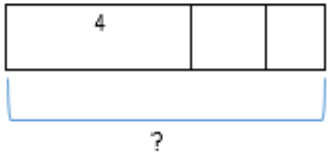
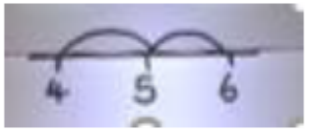
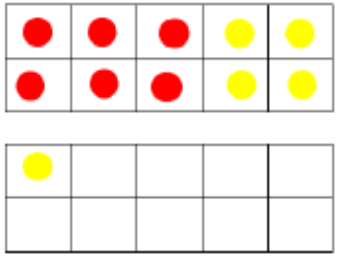
The Four Operations – Whole School Approach through CPA (Concrete, Pictorial, Abstract)

	EYFS	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Find a total number of items in two groups by counting all of them. Finding one more than a given number. Number bonds using quantities and objects to add two single numbers Counting on	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on- using cubes. Regrouping to make 100.	Adding three single digits. To combine two numbers.	Column method- regrouping. (up to 3 digits).	Column method- regrouping. (up to 4 digits)	Column method- regrouping. Use of place value counters for adding decimals.	Column method- regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.
Subtraction	Find one less from a group of objects rather than a number. Number bonds Counting back	Taking away ones Counting back Find the difference Part whole model Make 10 using the ten frame	Counting back Find the difference Part whole model Make 10	Column method with regrouping.	Column method with regrouping. (up to 4 digits)	Column method with regrouping. Abstract for whole numbers. Start with place value counters for decimals- with the same amount of decimal places.	Column method with regrouping. Abstract methods. Place value counters for decimals- with different amounts of decimal places.
Multiplicatio	Doubling	Recognising and making equal groups. Doubling Counting in multiples Use cubes, Numicon and other objects in the classroom	Arrays- showing commutative multiplication	Arrays 2d × 1d	Column multiplication- introduced with place value counters. (2 and 3 digit multiplied by 1 digit)	Column multiplication Abstract only but might need a repeat of year 4 first(up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication Abstract methods (multi-digit up to 4 digits by a 2 digit number) Decimals x 1 digit or integer
Division	Sharing objects into groups	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping Division within arrays- linking to multiplication Repeated subtraction	Division with a remainder-using lollipop sticks, times tables facts and repeated subtraction. 2d divided by 1d using base 10 or place value counters	Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the tenths and hundredths column.

Addition

Key language which should be used in the Primary Phase:

sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' , ' is the same as', grouping, regrouping , inverse operation, increase, number bonds, partition

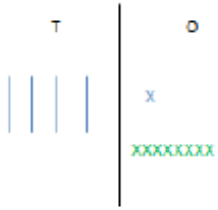
Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears etc)  		4 + 3 = 7 (four is a part, 3 is a part and the whole is seven) 
Counting on using number lines by using cubes or numicon  	A bar model which encourages the children to count on 	The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2? 4 + 2 
Regrouping to make 10 by using ten frames and counters/cubes or using numicon: 6 + 5  	Children to draw the ten frame and counters/cubes 	Children to develop an understanding of equality e.g 6 + ☐ = 11 and 6 + 5 = 5 + ☐ 6 + 5 = ☐ + 4

Addition – continued –

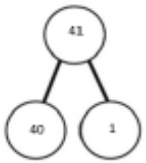
Base 10 or Dienes apparatus

TO + O using base 10. Continue to develop understanding of partitioning and place value
41 + 8

Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones.



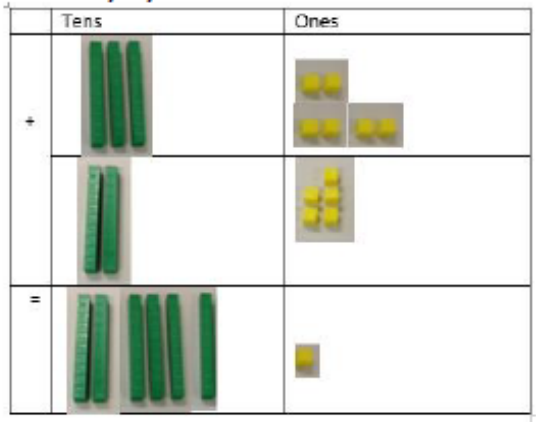
41 + 8



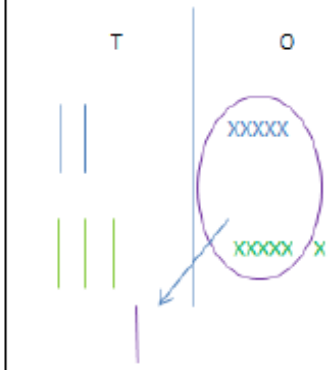
1 + 8 = 9
40 + 9 = 49

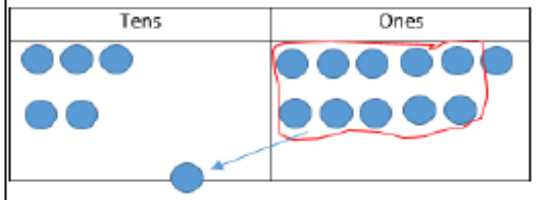
	4	1
+		8
	4	9

TO + TO using base 10. Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging. 36 + 25

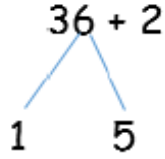


This could be done one of two ways:





Looking for ways to make 10



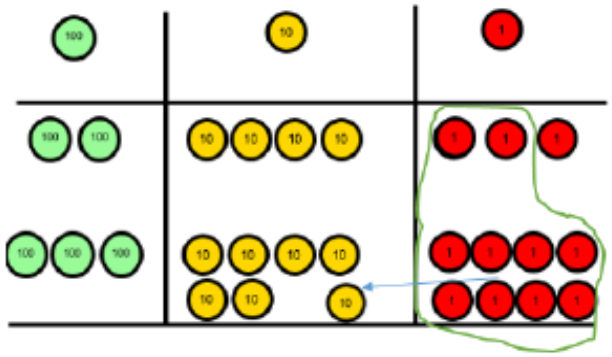
30 + 20 = 50
5 + 5 = 10
50 + 10 + 1 = 61

Formal method:

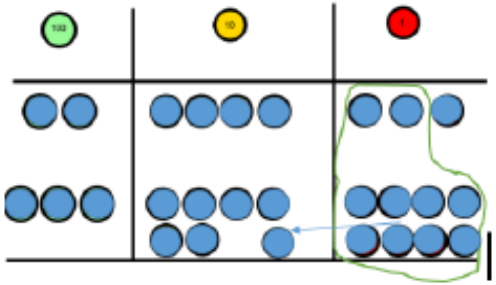
		1	
		3	6
+		2	5
		6	1

Addition continued

Use of place value counters to add HTO + TO, HTO + HTO etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract



Chidren to represent the counters e.g. like the image below



If the children are completing a word problem, draw a bar model to represent what it's asking them to do

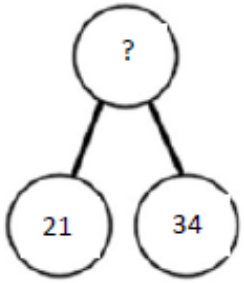
?	
243	368

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ } \\ 243 \\ +\ 368 \\ \hline 611 \end{array}$$

Leading to decimal addition

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ } \\ 2.4\ 3 \\ +\ 3.6\ 8 \\ \hline 6.1\ 1 \end{array}$$

Fluency variation, different ways to ask children to solve 21+34:



Sam saved £21 one week and £34 another. How much did he save in total?

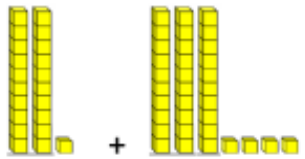
21+34=55. Prove it! (reasoning but the children need to be fluent in representing this)

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

21 + 34 =

$\square$ = 21 + 34

What's the sum of twenty one and thirty four?



Always use missing digit problems too:

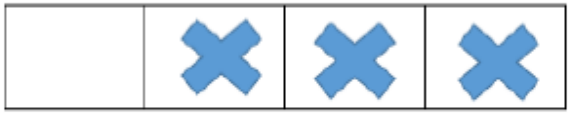
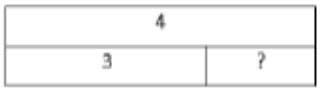
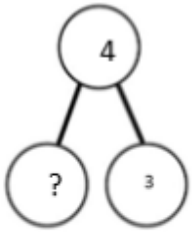
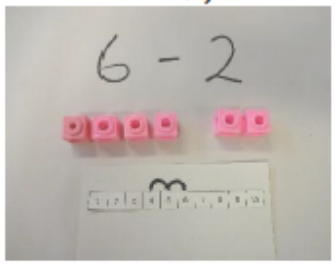
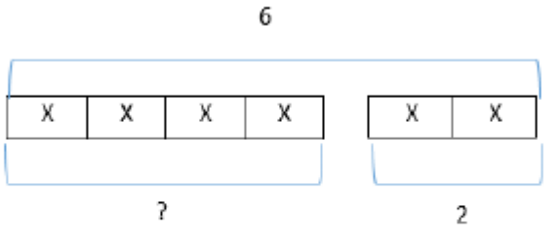
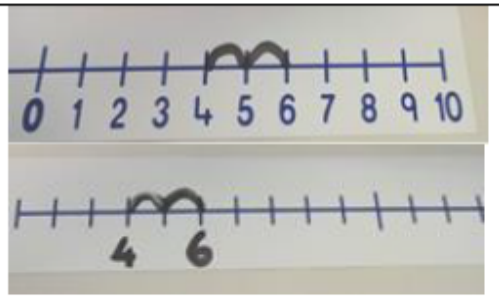
Tens	Ones
	?
?	4

Subtraction

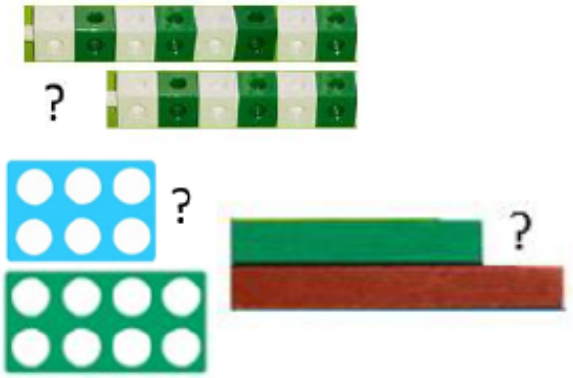
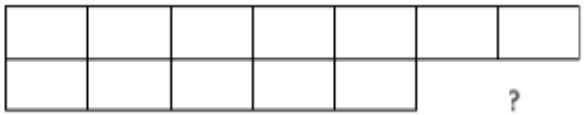
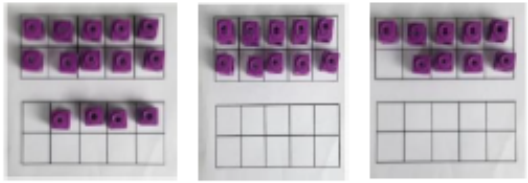
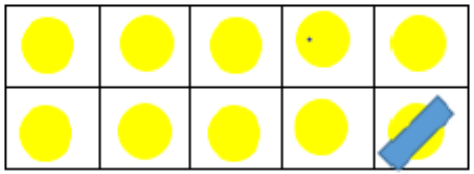
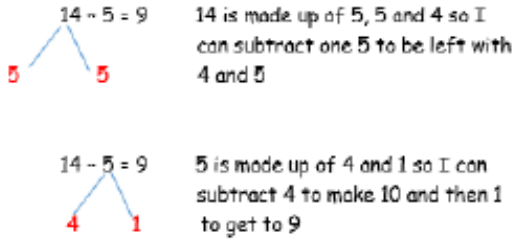
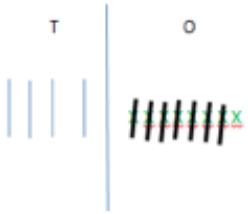
Key language which should be used in the Primary Phase:

take away, less than, the difference, subtraction, minus, fewer, decrease, inverse operation, number bonds,

'7 take away 3', ' the difference is 4', the difference between, exchange,

Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (use various objects too) rather than crossing out- children will physically remove the objects 4 − 3 = 1 	Children to draw the concrete resources they are using and cross out.  Use of the bar model: 	4- 3 = □ = 4 - 3  
Counting back (using number lines or number tracks) 	Children to represent what they see pictorially e.g. 	

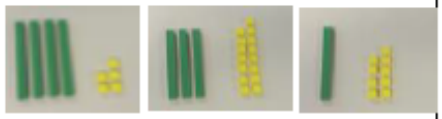
Subtraction continued

Finding the difference (using cubes, numicon or Cuisenaire rods, other objects can also be used) 	Children to draw the cubes/other concrete objects which they have used XXXXXXXX XXXXXX Use of the bar model 	Find the difference between 8 and 6. 8 - 6, the difference is ? Children to also explore why 9 - 7 = 8 - 6 (the difference, of each digit, has changed by 1 do the difference is the same- this will help when solving 10000-9987)
Making 10 (using numicon or ten frames) 14 - 5  Children could also do this by subtracting a 5 from the 10. 	Children to present the ten frame pictorially  	14 - 5 = 9 You also want children to see related facts e.g. 15 - 9 = 5 Children to represent how they have solved it e.g. 
Column method (using base 10) 48-7 		48 - 7 = 

Subtraction continued

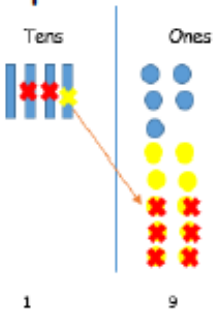
Column method (using base 10 and having to exchange)

45-26

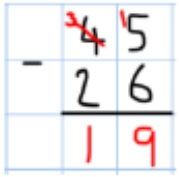


1) Start by partitioning 45
2) Exchange one ten for ten more ones
3) Subtract the ones, then the tens.

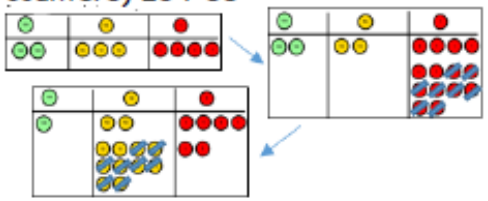
Represent the base 10 pictorially



It's crucial that the children understand that when they have exchanged the 10 they still have 45. 45 = 30 + 15

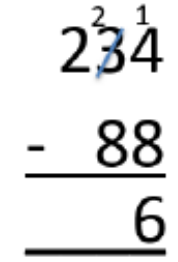


Column method (using place value counters) 234-88

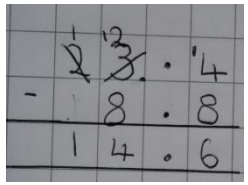


Once the children have had practice with the concrete, they should be able to apply it to any subtraction.

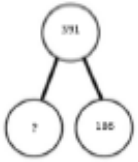
Like the other pictorial representations, children to represent the counters.

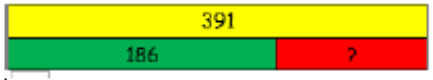


Using decimal places



Fluency variation, different ways to ask children to solve 391-186:





Raj spent £391, Timmy spent £186. How much more did Raj spend?

I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?

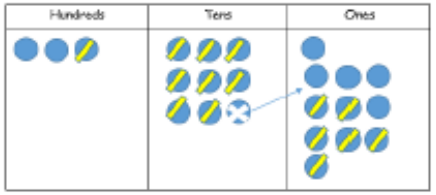
391 - 186

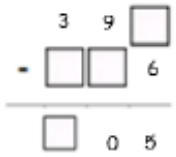
☐ = 391 - 186

391
-186
———

Find the difference ebtween 391 and 186
Subtract 186 from 391.
What is 186 less than 391?

What's the calculation? What's the answer?

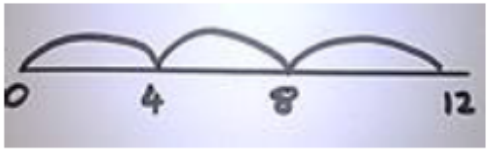
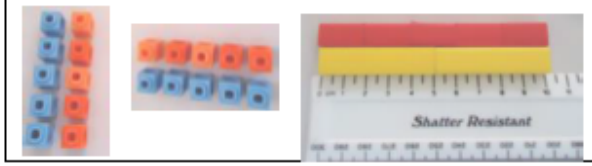
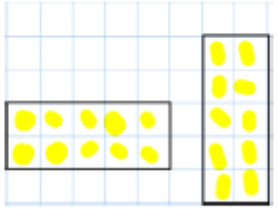


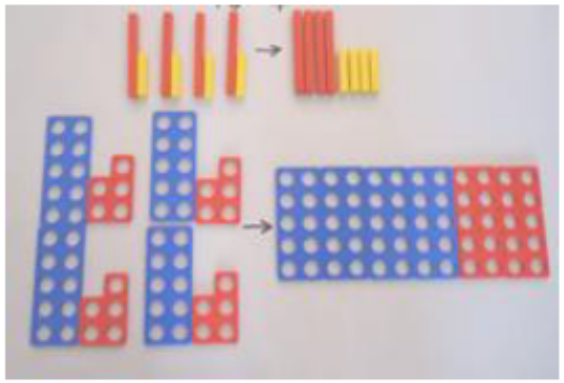
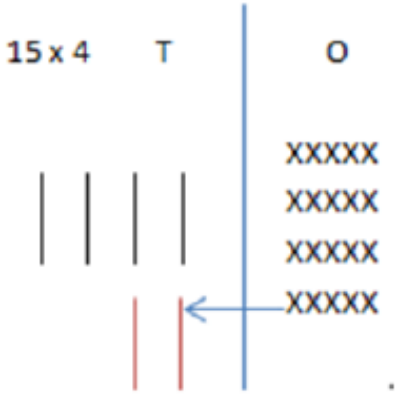
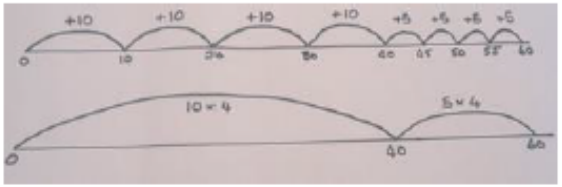


Multiplication

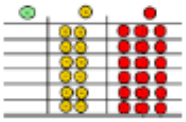
Key language which should be used:

Double, times, twice, multiplied by, the product of, product, groups of, arrays, lots of, repeated addition, repeated grouping, 'is the same as', multiple, factor, commutative law, distributive law, associative law, scaling, partition

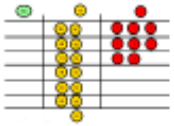
Concrete	Pictorial	Abstract
Repeated grouping/repeated addition (does not have to be restricted to cubes) 3 x 4 or 3 lots of 4 	Children to represent the practical resources in a picture e.g. XX XX XX XX XX XX Use of a bar model for a more structured method 	3 x 4 4 + 4 + 4
Use number lines to show repeated groups- 3 x 4 	Represent this pictorially alongside a number line e.g: 	Abstract number line **3 x 4 = 12** 
Use arrays to illustrate commutativity (counters and other objects can also be used) **2 x 5 = 5 x 2** 	Children to draw the arrays 	Children to be able to use an array to write a range of calculations e.g. 2 x 5 = 10 5 x 2 = 10 2 + 2 + 2 + 2 + 2 = 10 5 + 5 =10

Partition to multiply (use numicon, base 10, Cuisenaire rods) 4 x 15 	Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like: 	Children to be encouraged to show the steps they have taken 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used 
Formal column method with place value counters or base 10 (at the first stage- no exchanging) 3 x 23 Make 23, 3 times. See how many ones, then how many tens 	**Children to represent the counters in a pictorial way** 	Children to record what it is they are doing to show understanding 3 x 23 20 3 3 x 20 = 60 3 x 3 = 9 60 + 9= 69 23 x 3 69
Formal column method with place value counters (children need this stage, initially, to understand how the column method works)	Children to represent the counters/base 10, pictorially e.g. the image below.	**6 x 23** 6 x 3 = 18 6 x 20 = 120 120 + 18 = 138

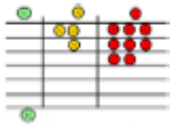
6 x 23



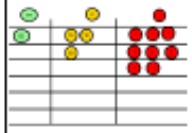
Step 1: get 6 lots of 23



Step 2: 6 x 3 is 18. Can I make an exchange? Yes! Ten ones for one ten….



Step 3: 6 x 2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred…



Step 4- what do I have I each column?

The aim is to get to the formal method but the children need to understand how it works.

6 x 23 =

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$$

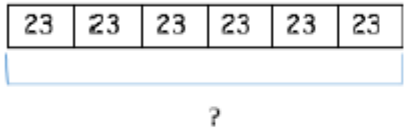
When children start to multiply 3d x 3d and 4d x 2d etc, they should be confident with the abstract:

To get 744 children have solved 6 x 124
To get 2480 they have solved 20 x 124

$$\begin{array}{rrrr} & 1 & 2 & 4 \\ \times & & 2 & 6 \\ \hline & {\scriptstyle 1}7 & {\scriptstyle 2}4 & 4 \\ 2 & 4 & 8 & 0 \\ \hline 3 & 2 & 2 & 4 \\ \hline 1 & 1 & & \end{array}$$

Answer: 3224

Fluency variation, different ways to ask children to solve 6 x 23:



With the counters, prove that 6 x 23 = 138

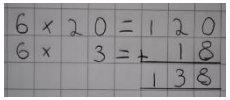
Why is 6 x 23 = 23 x 6?

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

Tom saved 23p three days a week. How much did he save in 2 weeks?

Find the product of 6 and 23

6 x 23 =

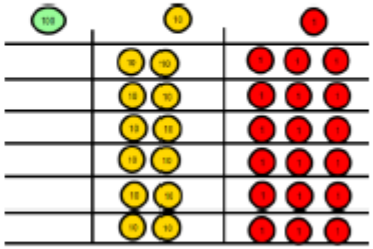


☐ = 6 x 23

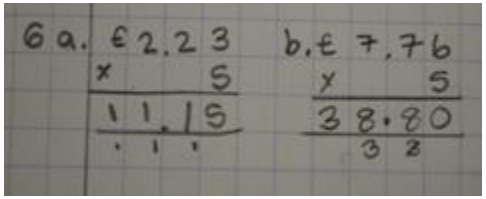
6 × 23

23 × 6

What's the calculation? What's the answer?

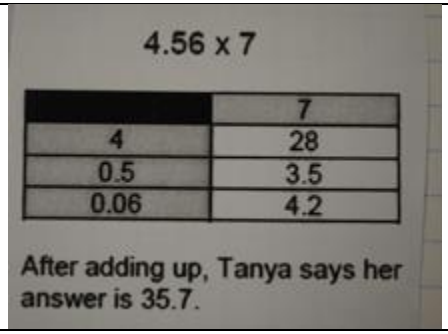


Multiply multi- digit numbers up to 4 digits
Using decimal places in formal written methods



Is the statement true or false?

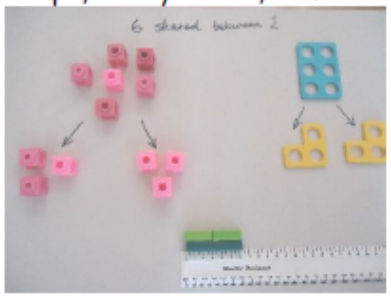
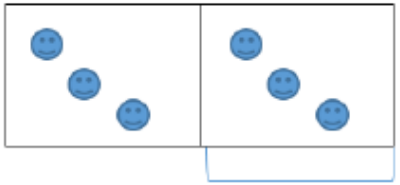
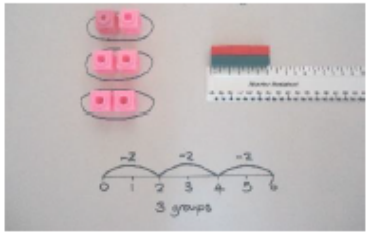
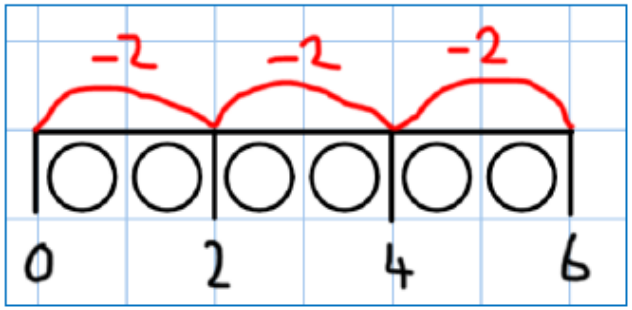
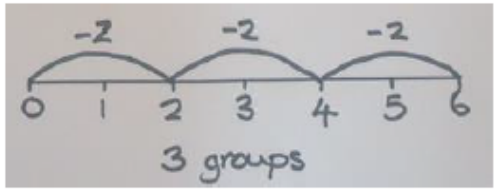
Using the grid method

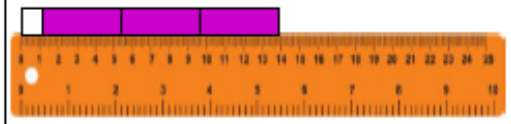
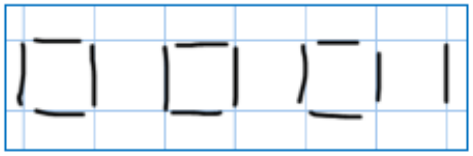
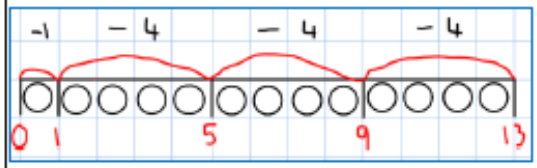
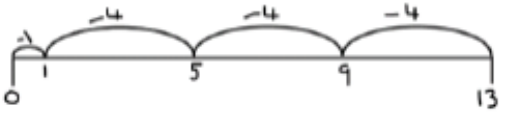
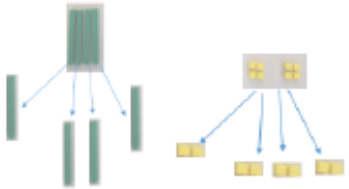
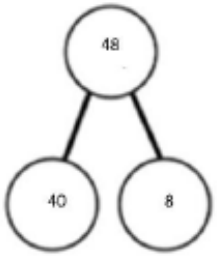
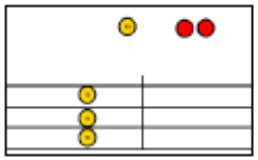
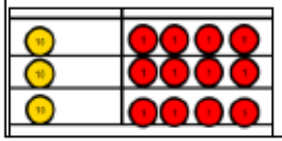
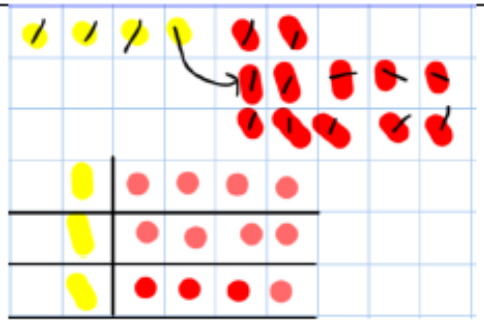


Division

Key language which should be used in the Primary Phase:

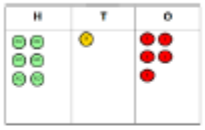
share, group, divide, divide by, half, 'is equal to', 'is the same as', remainders, repeated subtraction, repeated grouping, chunking, exchange, ratio, quotient, divisor, partition

Concrete	Pictorial	Abstract
6 shared between 2 (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates) 	 This can also be done in a bar so all 4 operations have a similar structure: 	6 ÷ 2 = 3 What's the calculation? 3 | 3
Understand division as repeated grouping and subtracting 6 ÷ 2 		Abstract number line 
2d ÷ 1d with remainders 13 ÷ 4 = 3 remainder 1	Children to have chance to represent the resources they use in a pictorial way e.g. see below:	13 ÷ 4 = 3 remainder 1 Children to count their times tables facts in their heads

Use of lollipop sticks to form wholes  Use of Cuisenaire rods and rulers (using repeated subtraction) 	 	
2d divided by 1d using base 10 (no remainders) SHARING **48 ÷ 4 = 12**  Start with the tens.	Children to represent the base 10 and sharing pictorially.	48 ÷ 4 4 tens ÷ 4 = 1 ten 8 ones ÷ 4 = 2 ones  10 + 2 = 12
Sharing using place value counters. 42 ÷ 3= 14  1. Make 42. Share the 4 tens between 3. Can we make an exchange with the extra 10?  Exchange the ten for 10 ones and share out 12 ones		42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Use of the 'bus stop method' using grouping and counters. Key language for grouping- how many groups of X can we make with X hundreds'- *this can also be done using sharing!*

615 ÷ 5



Step 1: make 615

Step 2: Circle your groups of 5

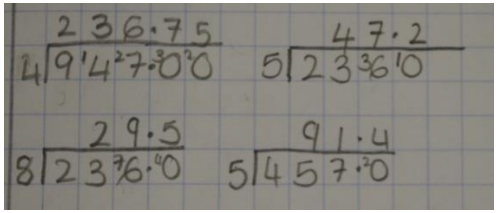
Step 3: Exchange 1H for 10T and circle groups of 5

Step 4: exchange 1T for 10ones and circles groups of 5

This can easily be represented pictorially until the pupils no longer need to do it. It can also be done to decimal places if you have a remainder!

$$5\overline{)615}\quad \begin{matrix}123\\ {}^{1}{}^{1}\end{matrix}$$

Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context



Fluency variation, different ways to ask children to solve 615 ÷ 5:

Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?

I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

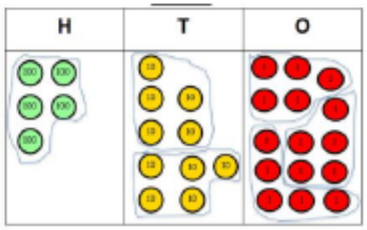
$$5\overline{)615}$$

615 ÷ 5 =

☐ = 615 ÷ 5

How many 5's go into 615?

What's the calculation? What's the answer?



Key Language

sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' , 'is the same as', grouping, regrouping , inverse operation, increase, take away, less than, the difference, comparing and associated language, whole number, integer, subtraction, minus, fewer, decrease, number bonds, calculation, operation, '7 take away 3', ' the difference is 4', the difference between, exchange, double, times, twice, multiplied by, the product of, product, groups of, arrays, lots of, repeated addition, 'is the same as', multiple, factor, commutative law, distributive law, associative law, scaling, compensation, partitioning, related facts, inverse operation, repeated grouping (aggregation), HTO – (hundreds, tens and ones and above~ up to millions), share, group, divide, divide by, half, remainders, repeated subtraction, chunking, exchange, ratio, quotient, divisor, dividend, remainder.

Types of Models

Arrays

Bar Model (Singapore Bar Model)

Part Whole Model

Formal Column addition

Formal Column subtraction

Grid method – (multiplication)

Formal column multiplication – long or short (compact)

Formal Division - Long and short methods (compact)